Geometry R – Mr. Bo
Unit 2B – Day 4 Notes

Name Notes
Date

Sequences of Rigid Motions

Composition - A sequence of 2 or more transformations applied to a shape.

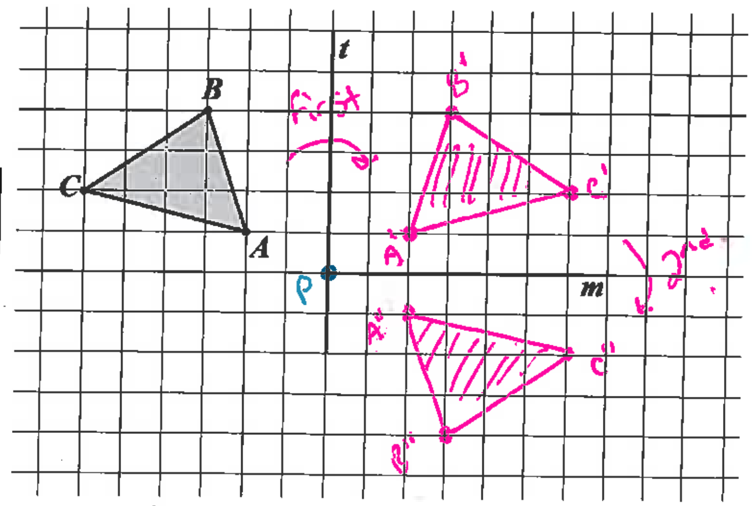
Isometry – A sequence of rigid motions that result in a congruent figure.

1a.

Composition Symbol

$$\underbrace{r_{line\,m}}_{\boxed{\text{Last}}} \circ \underbrace{r_{line\,t}}_{\boxed{\text{First}}}$$

* transformation on the right is always done first!



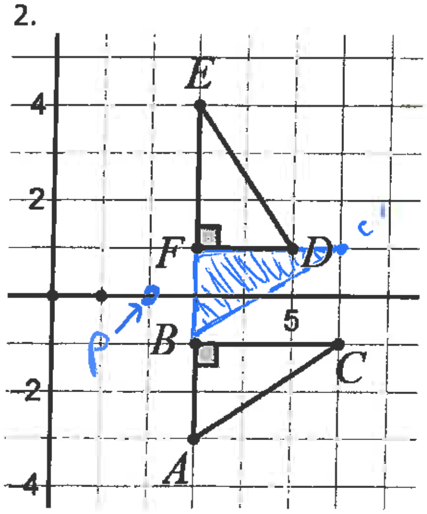
1b. Is the composition in part a an **Isometry**? Explain your reasoning.

yes. $\triangle A''B''C''$ is congruent (same size and shape) as $\triangle ABC$.

1c. In many cases, an *Isometry* can be simplified into a single rigid motion. Precisely describe a **single** rigid motion that would map ΔABC directly onto $\Delta A''B''C''$?

rotate counter clock wise around point P (where t and m intersect)

2.



a. Describe a precise sequence of 2 or more rigid motions which would map ΔABC onto ΔDFE.

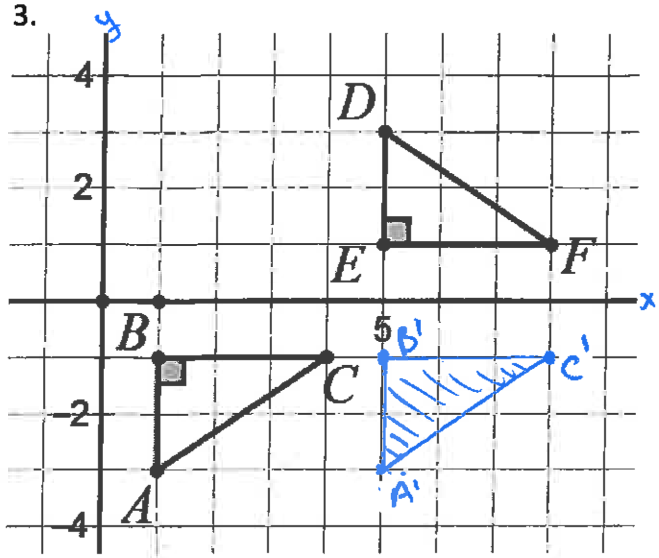
① translate $\triangle ABC$ 2 units up. So that B maps to F. (form $\triangle FBC'$)

② Rotate 90° counter-clock wise so C' maps to E.

b. Precisely describe a single rigid motion that would also map ΔABC onto ΔDFE.

Rotate 90° counter clock wise around point P.

3.



a. Precisely describe a sequence of rigid motions which would map ΔABC onto ΔDEF.

① translate ΔABC Right 4 units to make $\Delta A'B'C'$.

② Reflect $\Delta A'B'C'$ over the x-axis to form ΔDEF.

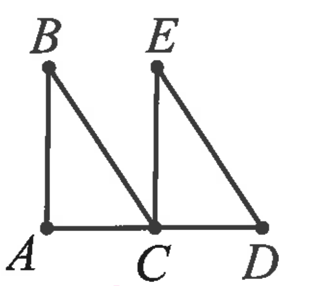
b. Is it possible to describe a single rigid motion that would map ΔABC onto ΔDEF? If so, describe precisely the transformation or if not, explain why not.

no, the composition of a reflection and translation can't be represented by a single rigid motion.

Using Rigid Motion to Justify Congruency

Theorem: If 2 figures in a plane are congruent, then there will always be a sequence of rigid motions that will map one figure onto the other.

4.



a. Precisely describe a single rigid motion that would map A onto C.

translate A along vector $\overrightarrow{AC}$ so that A maps to C.

b. Under this same rigid motion would C be mapped to D? Why or why not? Explain your reasoning.

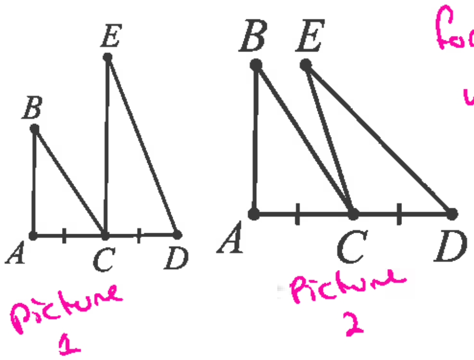
It might not. we don't know if the dist from C to D is the same as from A to C.

* we will assume we have no means for measuring in the picture.

c. In order for C to be mapped to D under this rigid motion, what additional information must be known about the picture?

we must know that $\overline{AC} \cong \overline{CD}$.

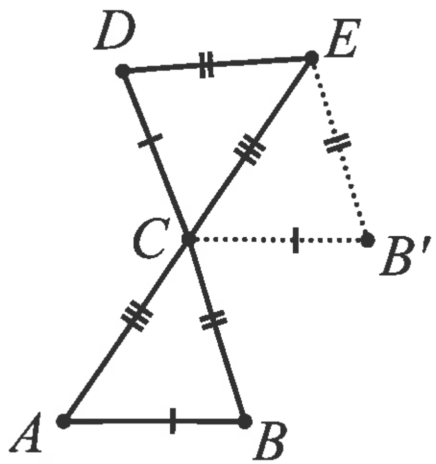
d. Now suppose $\overline{AC} \cong \overline{CD}$, so that C maps to D under this rigid motion. What additional information must be known for B to map to E? *Use these two pictures to help guide your reasoning.*



for B to map to E we would need $\boxed{\overline{AB} \cong \overline{CE}}$ (eliminates possibility of picture 1).

and we need to know $\boxed{\angle BAC \cong \angle ECD}$ (eliminating possibility of picture 2).

6. Precisely describe a sequence of rigid motions that maps ΔABC onto ΔCDE.



The 1st rigid motion maps ΔABC to $\Delta CB'E$.

a. Precisely describe the 1st Rigid Motion:

translate ΔABC along $\overrightarrow{AC}$ so that $A \rightarrow C$ and $C \rightarrow E$.

b. How do you know that C maps to E?

Since $\overline{AC} \cong \overline{CE}$, A and C will move the same distance.

The 2nd rigid motion maps $\Delta CB'E$ to ΔCDE.

c. Precisely describe the 2nd Rigid Motion:

reflect $\Delta CB'E$ over $\overline{CE}$ so that B' maps to D.

d. How do you know that B′ maps to D?

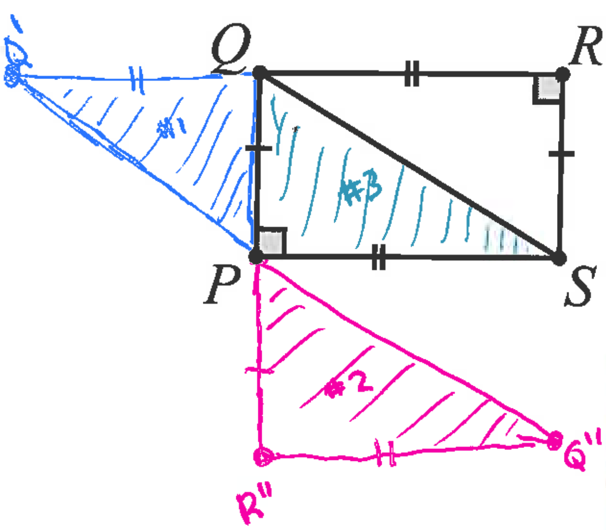
Since ∠ measure and segment lengths are preserved under translation and reflection, B' will map to D.

we will discuss this in more detail in another unit.

6e. How do you know that $\angle B \cong \angle D$?

∠ measure is preserved under translations + reflections.

7.



a. Precisely describe a sequence of rigid motions that maps ΔPQS onto ΔRSQ. *Sketch the resulting triangle for each rigid motion in the sequence.*

b. Explain how the sequence of rigid motions make $\angle RQS \cong \angle PSQ$.

1. Translate ΔQRS along $\overrightarrow{RQ}$ so that R maps to Q and S maps to P. to form $\Delta Q'QP$.
2. Rotate $\Delta Q'QP$ clockwise around P, 180° to form $\Delta Q''R''P$.
3. translate $\Delta Q''R''P$ along vector $\overrightarrow{PQ}$ so $\Delta Q''R''P$ maps to ΔSPQ.

*there are many other sequences that can be used.

→ $\angle RQS \cong \angle PSQ$ because translation + Rotation preserve angle measure.